

Application and Numerical Simulation of Modelled Epidemical Problems as Ordinary Differential Equation Using New Computation Method

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ABSTRACT

This study presents the development and application of a New Computational Method (NCM) for the numerical simulation of epidemiological problems modelled as ordinary differential equations. The method was derived using power series polynomial approximations combined with interpolation and collocation techniques at selected grid and off-grid points. The resulting scheme was formulated as a one-step block method capable of solving first-order epidemiological differential equations efficiently. Fundamental numerical properties of the proposed method were investigated, including order, consistency, zero-stability, convergence, and absolute stability. The analysis revealed that the method possesses an order of eleven, thereby ensuring high accuracy. Furthermore, the method was shown to be consistent, zero-stable, and convergent according to Dahlquist's convergence theorem. The region of absolute stability was obtained using the boundary locus technique, confirming the suitability of the method for the numerical integration of epidemiological models. To evaluate its performance, the NCM was applied to three representative epidemiological differential equation models, namely the Logistic Growth Model, the Decay Model and the Mixture Model. Numerical results obtained from the simulations were compared with exact analytical solutions and existing numerical methods reported in the literature. The computed solutions closely matched the exact solutions across all test problems and successfully reproduced the qualitative behaviour of the underlying models. The comparisons demonstrated that the NCM is highly reliable, efficient and robust for solving a wide range of epidemiological differential equations. Consequently, the NCM provides an effective computational tool for modelling and simulating disease-related and other applied mathematical phenomena governed by ordinary differential equations.

Keywords: New Computational Method (NCM); Epidemiological Model; Ordinary Differential Equations; Block Method; Numerical Simulation; Stability Analysis; Convergence; Logistic Model; Decay Model; Mixture Model; Ordinary Differential Equations.

1.0. Introduction

Numerical methods are essential for applied domains, which include scientific research, social science studies and engineering applications and medical practice, because exact solutions to their problems cannot be obtained. The methods enable various practical uses, which include spacecraft trajectory prediction and product safety testing and insurance risk assessment, through their combination with simulation methods that depend on solving PDEs [1]. Numerical analysts develop methods to find approximate solutions, which they use to create solutions that approximate precise results, but they must overcome difficulties which need either extensive computation or limited time to achieve precise outcomes [2-4]. Numerical methods find applications in various fields, which include multidimensional root-finding and industry forecasts and computing and profit-loss calculations, while their theoretical and practical applications extend to engineering and medical and business fields [5-7].

The 1927 SIR model developed by Kermack and McKendrick serves as a key example for epidemiologists who use differential equations to study infectious disease transmission. The model divides populations into three groups which include susceptible (S) individuals, infective (I) individuals, and recovered (R) individuals while using oscillatory differential equations to show how diseases progress [8]. Researchers create new numerical methods to study those models because they need better solutions than what existing methods can provide. The block hybrid methods use power series polynomials through interpolation and collocation to improve simulation accuracy and efficiency in epidemiological models which present difficulties because of their intricate disease spread patterns [8-11].

The Predictor-Corrector Method has several limitations that affect the numerical methods' efficiency and applicability in numerical computations. The method requires an initial prediction step which creates errors that spread through the entire computation process. The corrector needs multiple iterations to reach an accurate solution when the predictor delivers an incorrect result which results in higher computational expenses and longer processing durations. The problem occurs during stiff equation solution because inaccurate predictions lead to gradual convergence or complete method failure which reduces overall method stability [12]. The correction process uses implicit methods which require iterative methods like Newton's method. This requirement increases computational complexity especially in cases of nonlinear systems and systems that require high-dimensional processing [13].

The Predictor-Corrector Method faces its main difficulty because it requires multiple function evaluations during large-scale computations. The method needs to solve equations during both prediction and correction steps which results in higher processing requirements than fully implicit methods that reduce extra work. The system loses its operational value because it cannot operate effectively to meet real-time requirements because its calculation process needs to be completed quicker [14]. The process of selecting an appropriate step size proves essential for achieving both stability and accuracy while selecting an unsuitable step size results in unnecessary computation expenses and diminished accuracy. The implicit Runge-Kutta and Rosenbrock methods provide better stability and efficiency for solving stiff and large-scale problems than other numerical methods according to ref. [13]. The method's accuracy and effectiveness face limitations because some implementations depend on lower-order predictors according to [15]. The existing numerical techniques require additional development to achieve better performance across different disciplines according to ref. [16].

The Block Method was created as a new method to solve the problems that exist in the traditional Predictor-Corrector Method [17]. The numerical method divides the time period into smaller sections which allow it to solve different schemes at various off-grid locations throughout each time section. The Block Method uses implicit integration schemes which include backward Euler and Rosenbrock methods to handle stiff and highly nonlinear systems through balanced computational distribution [18]. The primary advantages of block method is to enhanced efficiency as it divide into smaller partitions to reduces the computational burden [2]. The Block Method solves both the computational and stability problems of the Predictor-Corrector Method while providing research opportunities to develop better numerical methods which can effectively solve complex differential equations.

This research considers the simulation of first order oscillatory differential equation of the form

$$\frac{dy}{dx} = f(x, y), y(0) = y_0, x \in [a, b] \quad (1)$$

where $f : \mathbb{R} \times \mathbb{R}^m \rightarrow \mathbb{R}^m$, $y, y_0 \in \mathbb{R}^m$, f satisfy the Lipchitz condition.

1.1. Study Objectives

The following are the objectives.

1. To develop a new computational method with multiple off-grid points using interpolation and collocation method,

2. To obtain the continuous form new computational method,
3. To analyse the basic properties of the computation method using order and error constant, consistency, convergent and zero-stability and stability region of the method,
4. To test the performance of the new computational method on some epidemical model using test example,
5. To shows the performance and comparison of the new computational method with the existing methods in literature.

2.0. Derivation of the New Computational Methods

The New Computational Method (NCM) derivation process in this section uses power series polynomial approximations to create a one-step solution method for the specific epidemic models that equation (1) defines. The method requires single-point power series interpolation together with multiple-point series collocation through alignment to partition points present in one interval.

The power series polynomial as defined by

$$y(x) = \sum_{i=0}^{\omega+\varpi-1} a_i x^i \quad (2)$$

The power series polynomial (2) is considered to simulate equation (1), for $x \in [x_n, x_{n+1}]$ where $\omega + \varpi$ are the number of interpolation and collocation, $n = 0, 1, 2, \dots, N-1$, a_i 's the determination of the real coefficients is integral to this process. Additionally, a constant step size $h = x_n - x_{n-1}$ for partitioning the interval $[a, b]$ is prescribed by the partition itself $\pi_N: a = x_0 < x_1 < x_2 < \dots < x_n < x_{n+1} < \dots < x_N = b$.

The new computational method was derived by using partitioned points

$$\left\{ 0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1 \right\}$$

Using equation (2) with $\omega = 1, \varpi = 11$, we have a polynomial as

$$y(x) = \sum_{i=0}^{11} a_i x^i \quad (3)$$

Differentiating (3) once, to get

$$y'(x) = \sum_{i=0}^{11} i a_i x^{i-1} \quad (4)$$

Substituting (4) and (3) into (1) gives

$$\sum_{i=0}^{11} a_i x^i = f(x) \quad (5)$$

$$\sum_{i=0}^{11} i a_i x^{i-1} = f(x, y) \quad (6)$$

Now interpolating (5) at $\omega = 1$ and collocating (6) at $\varpi = 0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1$.

This leads to a system of equations expressed in matrix form as

$$\begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 & x_{n+1}^9 & x_{n+1}^{10} & x_{n+1}^{11} \\ 0 & 1 & 2x_n & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 & 9x_{n+1}^8 & 10x_{n+1}^9 & 11x_{n+1}^{10} \\ 0 & 1 & 2x_{n+\frac{1}{10}} & 3x_{n+\frac{1}{10}}^2 & 4x_{n+\frac{1}{10}}^3 & 5x_{n+\frac{1}{10}}^4 & 6x_{n+\frac{1}{10}}^5 & 7x_{n+\frac{1}{10}}^6 & 8x_{n+\frac{1}{10}}^7 & 9x_{n+\frac{1}{10}}^8 & 10x_{n+\frac{1}{10}}^9 & 11x_{n+\frac{1}{10}}^{10} \\ 0 & 1 & 2x_{n+\frac{1}{5}} & 3x_{n+\frac{1}{5}}^2 & 4x_{n+\frac{1}{5}}^3 & 5x_{n+\frac{1}{5}}^4 & 6x_{n+\frac{1}{5}}^5 & 7x_{n+\frac{1}{5}}^6 & 8x_{n+\frac{1}{5}}^7 & 9x_{n+\frac{1}{5}}^8 & 10x_{n+\frac{1}{5}}^9 & 11x_{n+\frac{1}{5}}^{10} \\ 0 & 1 & 2x_{n+\frac{3}{10}} & 3x_{n+\frac{3}{10}}^2 & 4x_{n+\frac{3}{10}}^3 & 5x_{n+\frac{3}{10}}^4 & 6x_{n+\frac{3}{10}}^5 & 7x_{n+\frac{3}{10}}^6 & 8x_{n+\frac{3}{10}}^7 & 9x_{n+\frac{3}{10}}^8 & 10x_{n+\frac{3}{10}}^9 & 11x_{n+\frac{3}{10}}^{10} \\ 0 & 1 & 2x_{n+\frac{2}{5}} & 3x_{n+\frac{2}{5}}^2 & 4x_{n+\frac{2}{5}}^3 & 5x_{n+\frac{2}{5}}^4 & 6x_{n+\frac{2}{5}}^5 & 7x_{n+\frac{2}{5}}^6 & 8x_{n+\frac{2}{5}}^7 & 9x_{n+\frac{2}{5}}^8 & 10x_{n+\frac{2}{5}}^9 & 11x_{n+\frac{2}{5}}^{10} \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x_{n+\frac{1}{2}}^2 & 4x_{n+\frac{1}{2}}^3 & 5x_{n+\frac{1}{2}}^4 & 6x_{n+\frac{1}{2}}^5 & 7x_{n+\frac{1}{2}}^6 & 8x_{n+\frac{1}{2}}^7 & 9x_{n+\frac{1}{2}}^8 & 10x_{n+\frac{1}{2}}^9 & 11x_{n+\frac{1}{2}}^{10} \\ 0 & 1 & 2x_{n+\frac{3}{5}} & 3x_{n+\frac{3}{5}}^2 & 4x_{n+\frac{3}{5}}^3 & 5x_{n+\frac{3}{5}}^4 & 6x_{n+\frac{3}{5}}^5 & 7x_{n+\frac{3}{5}}^6 & 8x_{n+\frac{3}{5}}^7 & 9x_{n+\frac{3}{5}}^8 & 10x_{n+\frac{3}{5}}^9 & 11x_{n+\frac{3}{5}}^{10} \\ 0 & 1 & 2x_{n+\frac{7}{10}} & 3x_{n+\frac{7}{10}}^2 & 4x_{n+\frac{7}{10}}^3 & 5x_{n+\frac{7}{10}}^4 & 6x_{n+\frac{7}{10}}^5 & 7x_{n+\frac{7}{10}}^6 & 8x_{n+\frac{7}{10}}^7 & 9x_{n+\frac{7}{10}}^8 & 10x_{n+\frac{7}{10}}^9 & 11x_{n+\frac{7}{10}}^{10} \\ 0 & 1 & 2x_{n+\frac{4}{5}} & 3x_{n+\frac{4}{5}}^2 & 4x_{n+\frac{4}{5}}^3 & 5x_{n+\frac{4}{5}}^4 & 6x_{n+\frac{4}{5}}^5 & 7x_{n+\frac{4}{5}}^6 & 8x_{n+\frac{4}{5}}^7 & 9x_{n+\frac{4}{5}}^8 & 10x_{n+\frac{4}{5}}^9 & 11x_{n+\frac{4}{5}}^{10} \\ 0 & 1 & 2x_{n+\frac{9}{10}} & 3x_{n+\frac{9}{10}}^2 & 4x_{n+\frac{9}{10}}^3 & 5x_{n+\frac{9}{10}}^4 & 6x_{n+\frac{9}{10}}^5 & 7x_{n+\frac{9}{10}}^6 & 8x_{n+\frac{9}{10}}^7 & 9x_{n+\frac{9}{10}}^8 & 10x_{n+\frac{9}{10}}^9 & 11x_{n+\frac{9}{10}}^{10} \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 & 9x_{n+1}^8 & 10x_{n+1}^9 & 11x_{n+1}^{10} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \end{bmatrix} = \begin{bmatrix} y_{n+1} \\ f_n \\ f_{n+\frac{1}{10}} \\ f_{n+\frac{1}{5}} \\ f_{n+\frac{3}{10}} \\ f_{n+\frac{2}{5}} \\ f_{n+\frac{1}{2}} \\ f_{n+\frac{3}{5}} \\ f_{n+\frac{7}{10}} \\ f_{n+\frac{4}{5}} \\ f_{n+\frac{9}{10}} \\ f_{n+1} \end{bmatrix} \quad (7)$$

Employing the Gaussian elimination method on equation (7) yields the coefficients. These coefficients are subsequently substituted into equation (1) to obtain the implicit continuous method in the form:

$$y(x) = \alpha_1 y_{n+1} + h \left(\beta_0 f_{n+0} + \beta_{\frac{1}{10}} f_{n+\frac{1}{10}} + \beta_{\frac{1}{5}} f_{n+\frac{1}{5}} + \beta_{\frac{3}{10}} f_{n+\frac{3}{10}} + \beta_{\frac{2}{5}} f_{n+\frac{2}{5}} + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} + \beta_{\frac{3}{5}} f_{n+\frac{3}{5}} + \beta_{\frac{7}{10}} f_{n+\frac{7}{10}} + \beta_{\frac{4}{5}} f_{n+\frac{4}{5}} + \beta_{\frac{9}{10}} f_{n+\frac{9}{10}} + \beta_1 f_{n+1} \right) \quad (8)$$

$$\begin{aligned} \alpha_1 &= 1 \\ \beta_0 &= -\frac{16067}{598752} + t - \frac{7381}{504}t^2 + \frac{177133}{1512}t^3 - \frac{10511875}{18144}t^4 + \frac{8542325}{4536}t^5 - \frac{5369375}{1296}t^6 + \frac{4695625}{756}t^7 - \frac{9453125}{1512}t^8 \\ &+ \frac{6875000}{1701}t^9 + \frac{859375}{567}t^{10} + \frac{1562500}{6237}t^{11} \\ \beta_{\frac{1}{10}} &= -\frac{26575}{149688} + 50t^2 - \frac{149688}{189}t^3 + \frac{1997825}{504}t^4 - \frac{16598525}{1134}t^5 + \frac{3775625}{108}t^6 - \frac{20965625}{378}t^7 + \frac{1046875}{18}t^8 \\ &- \frac{65937500}{1701}t^9 + \frac{312500}{21}t^{10} + \frac{15625000}{6237}t^{11} \\ \beta_{\frac{1}{5}} &= \frac{16175}{199584} - \frac{225}{2}t^2 + \frac{153025}{84}t^3 - \frac{2898075}{224}t^4 + \frac{6619175}{128}t^5 - \frac{57981875}{432}t^6 + \frac{56396875}{252}t^7 - \frac{17546875}{72}t^8 \\ &+ \frac{94843750}{567}t^9 - \frac{4140625}{63}t^{10} + \frac{7812500}{693}t^{11} \\ \beta_{\frac{3}{10}} &= -\frac{5675}{12474} + 200t^2 - \frac{12474}{189}t^3 + \frac{10014475}{378}t^4 - \frac{21686950}{189}t^5 + \frac{8343125}{27}t^6 - \frac{33868750}{63}t^7 + \frac{5453125}{9}t^8 \\ &- \frac{242500000}{567}t^9 + \frac{32500000}{189}t^{10} + \frac{62500000}{2079}t^{11} \\ \beta_{\frac{2}{5}} &= \frac{4825}{11088} - \frac{525}{2}t^2 + \frac{168775}{36}t^3 - \frac{1792225}{48}t^4 + \frac{18214675}{108}t^5 - \frac{102070625}{216}t^6 + \frac{102070625}{126}t^7 - \frac{35640625}{36}t^8 \\ &+ \frac{19375000}{27}t^9 - \frac{19375000}{9}t^{10} + \frac{15625000}{297}t^{11} \\ \beta_{\frac{1}{2}} &= -\frac{17807}{24948} + 252t^2 - \frac{252}{3}t^3 + \frac{149625}{4}t^4 - \frac{1563725}{9}t^5 + \frac{27074375}{54}t^6 - \frac{58608125}{63}t^7 + \frac{10000000}{9}t^8 \end{aligned} \quad (9)$$

$$\begin{aligned}
 & -\frac{66875000}{81}t^9 + \frac{3125000}{9}t^{10} - \frac{6250000}{99}t^{11} \\
 \beta_3 = & \frac{4825}{561108800} - 175t^2 + \frac{174025}{54}t^3 - \frac{11544725}{432}t^4 + \frac{6835775}{54}t^5 - \frac{80666875}{216}t^6 + \frac{89384375}{126}t^7 - \frac{31234375}{36}t^8 \\
 & + \frac{5937500}{9}t^9 - \frac{7656250}{27}t^{10} + \frac{15625000}{297}t^{11} \\
 \beta_7 = & -\frac{5675}{12474} + \frac{600}{7}t^2 - \frac{600}{63}t^3 + \frac{600}{42}t^4 - \frac{12131950}{189}t^5 + \frac{5200625}{27}t^6 - \frac{1118750}{3}t^7 + \frac{29328125}{63}t^8 - \frac{29328125}{567}t^9 \\
 & + \frac{10000000}{63}t^{10} - \frac{62500000}{2079}t^{11} \\
 \beta_4 = & \frac{1562500}{199584} - \frac{225}{8}t^2 + \frac{88325}{168}t^3 - \frac{996675}{224}t^4 + \frac{996675}{504}t^5 - \frac{28405625}{432}t^6 + \frac{28405625}{252}t^7 - \frac{11828125}{72}t^8 \\
 & + \frac{73750000}{567}t^9 - \frac{73750000}{63}t^{10} + \frac{7812500}{693}t^{11} \\
 \beta_9 = & -\frac{26575}{149688} + \frac{50}{9}t^2 - \frac{6575}{63}t^3 + \frac{4033825}{4536}t^4 - \frac{4943525}{1134}t^5 + \frac{4341875}{324}t^6 - \frac{10090625}{378}t^7 + \frac{1859375}{54}t^8 \\
 & + \frac{1859375}{1701}t^9 - \frac{7187500}{567}t^{10} + \frac{15625000}{6232}t^{11} \\
 \beta_1 = & -\frac{16067}{598752} + \frac{1}{2}t^2 - \frac{7129}{756}t^3 + \frac{162875}{2016}t^4 - \frac{226150}{567}t^5 + \frac{59375}{48}t^6 - \frac{1883125}{756}t^7 + \frac{78125}{24}t^8 \\
 & - \frac{4531250}{1701}t^9 + \frac{78125}{63}t^{10} + \frac{1562500}{6232}t^{11}
 \end{aligned}$$

and

$$t = \frac{X - X_n}{h} \quad (10)$$

Simplify equation (8) at all the grid points to obtain the new Block Method (NCM) as

$$\begin{aligned}
 y_{n+\frac{1}{10}} = & y_n + \frac{26842253}{958003200}h f_n + \frac{164046413}{1197504000}h f_{n+\frac{1}{10}} - \frac{296725183}{1596672000}h f_{n+\frac{1}{5}} + \frac{12051709}{39916800}h f_{n+\frac{3}{10}} - \frac{33765029}{88704000}h f_{n+\frac{2}{5}} \\
 & + \frac{2227571}{6237000}h f_{n+\frac{1}{2}} - \frac{21677723}{88704000}h f_{n+\frac{3}{5}} + \frac{23643791}{199584000}h f_{n+\frac{7}{10}} - \frac{12318413}{319334400}h f_{n+\frac{4}{5}} + \frac{9071219}{1197504000}h f_{n+\frac{9}{10}} - \frac{3250433}{4790016000}h f_{n+1} \\
 y_{n+\frac{1}{5}} = & y_n + \frac{2046263}{74844000}h f_n + \frac{645431}{3742200}h f_{n+\frac{1}{10}} - \frac{2149811}{24948000}h f_{n+\frac{1}{5}} + \frac{355583}{1559250}h f_{n+\frac{3}{10}} - \frac{1258463}{4158000}h f_{n+\frac{2}{5}} + \frac{904403}{3118500}h f_{n+\frac{1}{2}} \\
 & - \frac{166931}{831600}h f_{n+\frac{3}{5}} + \frac{21833}{222750}h f_{n+\frac{7}{10}} - \frac{800243}{24948000}h f_{n+\frac{4}{5}} + \frac{118291}{18711000}h f_{n+\frac{9}{10}} - \frac{8501}{14968800}h f_{n+1} \\
 y_{n+\frac{3}{10}} = & y_n + \frac{108223}{3942400}h f_n + \frac{840607}{4928000}h f_{n+\frac{1}{10}} - \frac{879183}{19712000}h f_{n+\frac{1}{5}} + \frac{762497}{2464000}h f_{n+\frac{3}{10}} - \frac{670233}{1971200}h f_{n+\frac{2}{5}} + \frac{24399}{77000}h f_{n+\frac{1}{2}} \\
 & - \frac{2136259}{9856000}h f_{n+\frac{3}{5}} + \frac{259071}{2464000}h f_{n+\frac{7}{10}} - \frac{675441}{19712000}h f_{n+\frac{4}{5}} + \frac{6637}{985600}h f_{n+\frac{9}{10}} - \frac{11899}{19712000}h f_{n+1} \\
 y_{n+\frac{2}{5}} = & y_n + \frac{64121}{2338875}h f_n + \frac{400138}{2338875}h f_{n+\frac{1}{10}} - \frac{2159}{44550}h f_{n+\frac{1}{5}} + \frac{39752}{111375}h f_{n+\frac{3}{10}} - \frac{23423}{86625}h f_{n+\frac{2}{5}} + \frac{230788}{779625}h f_{n+\frac{1}{2}} \\
 & - \frac{17879}{86625}h f_{n+\frac{3}{5}} + \frac{2248}{22275}h f_{n+\frac{7}{10}} - \frac{25754}{779625}h f_{n+\frac{4}{5}} + \frac{15226}{2338875}h f_{n+\frac{9}{10}} - \frac{547}{935550}h f_{n+1} \\
 y_{n+\frac{1}{2}} = & y_n + \frac{1051285}{38320128}h f_n + \frac{1636625}{9580032}h f_{n+\frac{1}{10}} - \frac{599275}{12773376}h f_{n+\frac{1}{5}} + \frac{558725}{1596672}h f_{n+\frac{3}{10}} - \frac{461275}{2128896}h f_{n+\frac{2}{5}} + \frac{17807}{49896}h f_{n+\frac{1}{2}} \\
 & - \frac{465125}{2128896}h f_{n+\frac{3}{5}} + \frac{167675}{1596672}h f_{n+\frac{7}{10}} - \frac{62275}{1824768}h f_{n+\frac{4}{5}} + \frac{64175}{9580032}h f_{n+\frac{9}{10}} - \frac{22997}{38320128}h f_{n+1} \\
 y_{n+\frac{3}{5}} = & y_n + \frac{1689}{61600}h f_n + \frac{13169}{77000}h f_{n+\frac{1}{10}} - \frac{14787}{308000}h f_{n+\frac{1}{5}} + \frac{1363}{3850}h f_{n+\frac{3}{10}} - \frac{35229}{154000}h f_{n+\frac{2}{5}} + \frac{16083}{38500}h f_{n+\frac{1}{2}} \\
 & - \frac{25373}{154000}h f_{n+\frac{3}{5}} + \frac{1887}{19250}h f_{n+\frac{7}{10}} - \frac{2007}{61600}h f_{n+\frac{4}{5}} + \frac{71}{11000}h f_{n+\frac{9}{10}} - \frac{179}{308000}h f_{n+1} \\
 y_{n+\frac{7}{10}} = & y_n + \frac{18775351}{684288000}h f_n + \frac{5843887}{34214400}h f_{n+\frac{1}{10}} - \frac{10669897}{228096000}h f_{n+\frac{1}{5}} + \frac{9973607}{28512000}h f_{n+\frac{3}{10}} - \frac{2767667}{12672000}h f_{n+\frac{2}{5}} \\
 & + \frac{353633}{891000}h f_{n+\frac{1}{2}} - \frac{241129}{2534400}h f_{n+\frac{3}{5}} + \frac{4148249}{28512000}h f_{n+\frac{7}{10}} - \frac{8312311}{228096000}h f_{n+\frac{4}{5}} + \frac{1190357}{171072000}h f_{n+\frac{9}{10}} - \frac{84427}{136857600}h f_{n+1} \\
 y_{n+\frac{4}{5}} = & y_n + \frac{12818}{467775}h f_n + \frac{400448}{2338875}h f_{n+\frac{1}{10}} - \frac{38176}{779625}h f_{n+\frac{1}{5}} + \frac{278272}{779625}h f_{n+\frac{3}{10}} - \frac{12184}{51975}h f_{n+\frac{2}{5}} + \frac{330368}{779625}h f_{n+\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{34432}{259875} h f_{n+\frac{3}{5}} + \frac{176896}{779625} h f_{n+\frac{7}{10}} - \frac{3998}{779625} h f_{n+\frac{4}{5}} + \frac{2368}{467775} h f_{n+\frac{9}{10}} - \frac{1184}{2338875} h f_{n+1} \\
 y_{n+\frac{9}{10}} = & y_n + \frac{542331}{19712000} h f_n + \frac{837567}{4928000} h f_{n+\frac{1}{10}} - \frac{167427}{3942400} h f_{n+\frac{1}{5}} + \frac{829089}{2464000} h f_{n+\frac{3}{10}} - \frac{1880253}{9856000} h f_{n+\frac{2}{5}} + \frac{27459}{77000} h f_{n+\frac{1}{2}} \\
 & - \frac{537219}{9856000} h f_{n+\frac{3}{5}} + \frac{10773}{70400} h f_{n+\frac{7}{10}} - \frac{2065743}{19712000} h f_{n+\frac{4}{5}} + \frac{199809}{4928000} h f_{n+\frac{9}{10}} - \frac{4671}{3942400} h f_{n+1} \\
 y_{n+1} = & y_n + \frac{16067}{598752} h f_n + \frac{26575}{149688} h f_{n+\frac{1}{10}} - \frac{16175}{199584} h f_{n+\frac{1}{5}} + \frac{5675}{12474} h f_{n+\frac{3}{10}} - \frac{4825}{11088} h f_{n+\frac{2}{5}} + \frac{17807}{24948} h f_{n+\frac{1}{2}} \\
 & - \frac{4825}{11088} h f_{n+\frac{3}{5}} + \frac{5675}{12474} h f_{n+\frac{7}{10}} + \frac{16175}{199584} h f_{n+\frac{4}{5}} + \frac{26575}{149688} h f_{n+\frac{9}{10}} + \frac{16067}{598752} h f_{n+1}
 \end{aligned} \tag{11}$$

3.0. Analysis of the Basic Properties of New Computational Method (NCM)

The analysis of the basic properties of NCM are analyzed numerically, such as order and error constant, consistency, zero-stability and the region of absolute stability.

3.1. Order and Error Constant of NCM

The order and error constant of NCM was carryout according to ref. [7], by using

$$\begin{aligned}
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{1}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{164046413}{1197504000} \left(\frac{1}{10}\right)^j - \frac{296725183}{1596672000} \left(\frac{1}{5}\right)^j + \frac{12051709}{39916800} \left(\frac{3}{10}\right)^j - \frac{33765029}{88704000} \left(\frac{2}{5}\right)^j + \frac{2227571}{6237000} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{21677723}{88704000} \left(\frac{3}{5}\right)^j + \frac{23643791}{199584000} \left(\frac{7}{10}\right)^j - \frac{12318413}{319334400} \left(\frac{4}{5}\right)^j + \frac{9071219}{1197504000} \left(\frac{9}{10}\right)^j - \frac{3250433}{4790016000} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{1}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{645431}{3742200} \left(\frac{1}{10}\right)^j - \frac{2149811}{24948000} \left(\frac{1}{5}\right)^j + \frac{355583}{1559250} \left(\frac{3}{10}\right)^j - \frac{1258463}{4158000} \left(\frac{2}{5}\right)^j + \frac{904403}{3118500} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{166931}{831600} \left(\frac{3}{5}\right)^j + \frac{21833}{222750} \left(\frac{7}{10}\right)^j - \frac{800243}{24948000} \left(\frac{4}{5}\right)^j + \frac{118291}{18711000} \left(\frac{9}{10}\right)^j - \frac{8501}{14968800} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{3}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{840607}{4928000} \left(\frac{1}{10}\right)^j - \frac{879183}{19712000} \left(\frac{1}{5}\right)^j + \frac{762497}{2464000} \left(\frac{3}{10}\right)^j - \frac{670233}{1971200} \left(\frac{2}{5}\right)^j + \frac{24399}{77000} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{2136259}{9856000} \left(\frac{3}{5}\right)^j + \frac{259071}{2464000} \left(\frac{7}{10}\right)^j - \frac{675441}{19712000} \left(\frac{4}{5}\right)^j + \frac{6637}{985600} \left(\frac{9}{10}\right)^j - \frac{11899}{19712000} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{2}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{400138}{2338875} \left(\frac{1}{10}\right)^j - \frac{2159}{44550} \left(\frac{1}{5}\right)^j + \frac{39752}{111375} \left(\frac{3}{10}\right)^j - \frac{23423}{86625} \left(\frac{2}{5}\right)^j + \frac{230788}{779625} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{17879}{86625} \left(\frac{3}{5}\right)^j + \frac{2248}{22275} \left(\frac{7}{10}\right)^j - \frac{25754}{779625} \left(\frac{4}{5}\right)^j + \frac{15226}{2338875} \left(\frac{9}{10}\right)^j - \frac{547}{935550} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{1}{2}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{1636625}{9580032} \left(\frac{1}{10}\right)^j - \frac{599275}{12773376} \left(\frac{1}{5}\right)^j + \frac{558725}{1596672} \left(\frac{3}{10}\right)^j - \frac{461275}{2128896} \left(\frac{2}{5}\right)^j + \frac{17807}{49896} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{465125}{2128896} \left(\frac{3}{5}\right)^j + \frac{167675}{1596672} \left(\frac{7}{10}\right)^j - \frac{62275}{1824768} \left(\frac{4}{5}\right)^j + \frac{64175}{9580032} \left(\frac{9}{10}\right)^j - \frac{22997}{38320128} h f_{n+1} \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{3}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{13169}{77000} \left(\frac{1}{10}\right)^j - \frac{14787}{308000} \left(\frac{1}{5}\right)^j + \frac{1363}{3850} \left(\frac{3}{10}\right)^j - \frac{35229}{154000} \left(\frac{2}{5}\right)^j + \frac{16083}{38500} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{25373}{154000} \left(\frac{3}{5}\right)^j + \frac{1887}{19250} \left(\frac{7}{10}\right)^j - \frac{2007}{61600} \left(\frac{4}{5}\right)^j + \frac{71}{11000} \left(\frac{9}{10}\right)^j - \frac{179}{308000} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{7}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{5843887}{34214400} \left(\frac{1}{10}\right)^j - \frac{10669897}{228096000} \left(\frac{1}{5}\right)^j + \frac{9973607}{28512000} \left(\frac{3}{10}\right)^j - \frac{2767667}{12672000} \left(\frac{2}{5}\right)^j + \frac{353633}{891000} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{241129}{2534400} \left(\frac{3}{5}\right)^j + \frac{4148249}{28512000} \left(\frac{7}{10}\right)^j - \frac{8312311}{228096000} \left(\frac{4}{5}\right)^j + \frac{1190357}{171072000} \left(\frac{9}{10}\right)^j - \frac{84427}{136857600} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{4}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{400448}{2338875} \left(\frac{1}{10}\right)^j - \frac{38176}{779625} \left(\frac{1}{5}\right)^j + \frac{278272}{779625} \left(\frac{3}{10}\right)^j - \frac{12184}{51975} \left(\frac{2}{5}\right)^j + \frac{330368}{779625} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{34432}{259875} \left(\frac{3}{5}\right)^j + \frac{176896}{779625} \left(\frac{7}{10}\right)^j - \frac{3998}{779625} \left(\frac{4}{5}\right)^j + \frac{2368}{467775} \left(\frac{9}{10}\right)^j - \frac{1184}{2338875} (1)^j \right] \right. \\
 & \left[\sum_{j=0}^{\infty} \frac{\left(\frac{9}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_{n+j+1} \left[\frac{837567}{4928000} \left(\frac{1}{10}\right)^j - \frac{167427}{3942400} \left(\frac{1}{5}\right)^j + \frac{829089}{2464000} \left(\frac{3}{10}\right)^j - \frac{1880253}{9856000} \left(\frac{2}{5}\right)^j + \frac{27459}{77000} \left(\frac{1}{2}\right)^j \right. \right. \\
 & \left. \left. - \frac{537219}{9856000} \left(\frac{3}{5}\right)^j + \frac{10773}{70400} \left(\frac{7}{10}\right)^j - \frac{2065743}{19712000} \left(\frac{4}{5}\right)^j + \frac{199809}{4928000} \left(\frac{9}{10}\right)^j - \frac{4671}{3942400} (1)^j \right] \right] = 0
 \end{aligned} \tag{12}$$

$$\sum_{j=0}^{\infty} \frac{(1)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\begin{array}{c} \frac{26575}{149688} \left(\frac{1}{10}\right)^j - \frac{16175}{199584} \left(\frac{1}{5}\right)^j + \frac{5675}{12474} \left(\frac{3}{10}\right)^j - \frac{4825}{11088} \left(\frac{2}{5}\right)^j + \frac{17807}{24948} \left(\frac{1}{2}\right)^j \\ - \frac{4825}{11088} \left(\frac{3}{5}\right)^j + \frac{5675}{12474} \left(\frac{7}{10}\right)^j + \frac{16175}{199584} \left(\frac{4}{5}\right)^j + \frac{26575}{149688} \left(\frac{9}{10}\right)^j + \frac{16067}{598752} (1)^j \end{array} \right]$$

Thus, $\bar{C}_0 = 0, \bar{C}_1 = 0, \bar{C}_2 = 0, \bar{C}_3 = 0, \dots = 0, \bar{C}_p = 0$; implying that the order of NCM is 11.

That is, NCM is of order eleven, with the error constant given as

$$C_{12} = \begin{bmatrix} 1.2343 \times 10^{-14} & 1.7861 \times 10^{-14} & 1.8763 \times 10^{-14} & 1.8821 \times 10^{-14} & 1.2301 \times 10^{-14} \\ 1.2226 \times 10^{-14} & 1.0752 \times 10^{-14} & 1.9328 \times 10^{-14} & 1.5213 \times 10^{-14} & 1.4439 \times 10^{-14} \end{bmatrix}^T$$

3.2. Consistency of NCM

According to numerical analyst, NCM is consistent, since the order is greater than one [6]

3.3. Zero Stability of NCM

According to [6], the NCM is said to be zero-stable if as $h \rightarrow 0$, the root $z_j, j = 1(1)k$ of the first characteristic polynomial $\rho(z) = 0$ that is

$$\rho(z) = \det \left[\sum_{j=0}^k A^{(j)} z^{k-j} \right] = 0 \quad (13)$$

satisfies $|z_j| \leq 1$ and for those roots with $|z_j| = 1$, multiplicity must not exceed two. The NCM is expressed in the form

$$\rho(z) = z \left[\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right] = z^9(z-1) \quad (14)$$

Thus, solving for z in

$$\rho(z) = z^9(z-1) = 0 \quad (15)$$

Gives $z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 0, z_5 = 0, z_6 = 0, z_7 = 0, z_8 = 0, z_9 = 0$ and $z_{10} = 1$

Therefore, NCM is zero-stable.

3.4. Convergence of NCM

Based on Dahlquist's Theorem which state that, the requisite and complete prerequisites for a NCM to achieve convergence entail both consistency and zero-stability [19].

Therefore, the NCM is convergence since it is consistency and zero-stability.

3.5. Absolute Stability Region of NCM

The absolute stability region describes the region of complex z -plane. The boundary locus method was used on the NCM using [11] to obtain the stability polynomial as

$$\bar{h}(w) = h^{10} \left(\frac{127538035263931}{7449432883200000} w^9 - \frac{58777}{4204890} w^{10} \right) - h^9 \left(-\frac{1021842064371587}{6337191168000000000000000} w^9 + \frac{183517}{4752} w^{10} \right) + \left. \begin{aligned} &h^8 \left(-\frac{1240787094942223}{405580234752000000000000} w^9 + \frac{66528}{1181437} w^{10} \right) + h^7 \left(\frac{13387971131347}{1469604310478438400000} w^9 - \frac{1303}{3214155168} w^{10} \right) + \\ &h^6 \left(-\frac{2675675852951287}{979736206985625600000} w^9 + \frac{502211745}{4523} w^{10} \right) + h^5 \left(\frac{21747803594142223}{1216740704256000000000000} w^9 - \frac{985}{5629856} w^{10} \right) + \\ &h^4 \left(-\frac{155422825589737}{5431878144000000000000000} w^9 - \frac{986}{74640} w^{10} \right) + h^3 \left(\frac{161809957779906749}{391095226368000000000000} w^9 - \frac{867}{90875} w^{10} \right) + \\ &h^2 \left(\frac{14411557729}{189684633600000000000000} w^9 + \frac{332}{8524} w^{10} \right) + h \left(\frac{2400000}{10886400} w^9 - \frac{1}{8} w^{10} \right) - w^8 + w^9 \end{aligned} \right\} \quad (16)$$

The stability polynomial (3.5) is used to plot the absolute stability region as

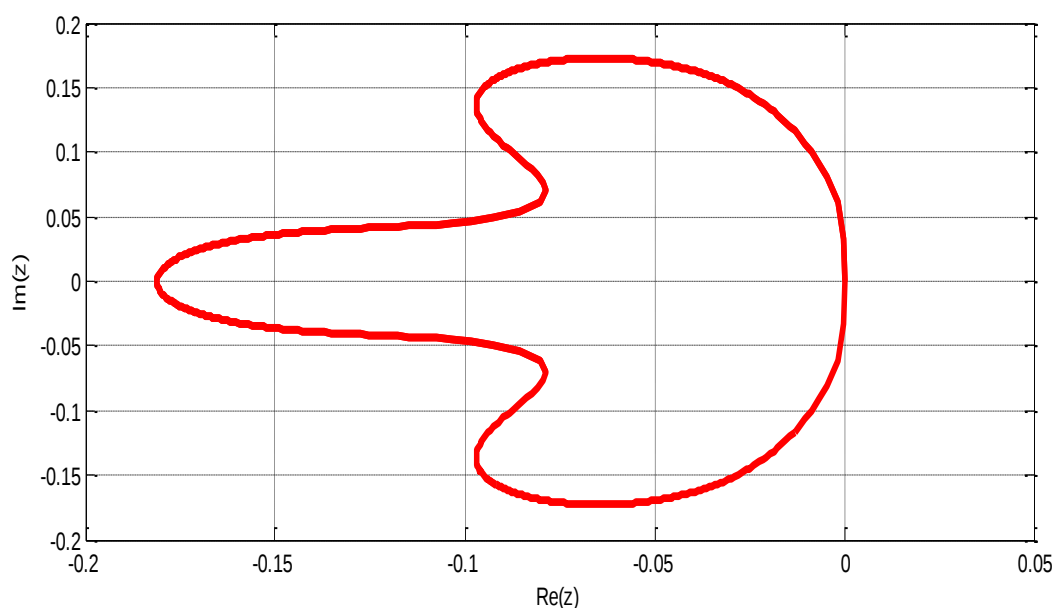


Figure 1. An A_{α} – Stability region of NCM.

4.0. Numerical Experiments

The NCM was numerically simulated on several epidemiological modelled differential equations of the form (1) as outlined below. The following notations shall be used in Tables below. The following notations shall be used in Tables 1 to 3 below.

t Point of Evaluation

ENCM means Error in New Computational Method

E[18] means Error in ref. [18]

E[21] means Error in ref. [21]

E[22] means Error in ref. [22]

E[23] means Error in in ref. [23]

Problem 1:

Consider the Logistic epidemiological differential equations Modelled as the logistic model (an extension of growth model) is the law that regulates with good estimation, the growth rate of a certain number of populations as function of time. The model is established on Assuming the population grows within a confined environment where resources are limited and there is no migration in or out, the change in population over time can be described mathematically. Let $x(t)$ be the population size at time t , the governing principle of its growth can be formulated as a differential equation

$$\frac{dx}{dT} = rx \left(1 - \frac{x}{k} \right) \quad (17)$$

Where $k > 0$ is the carrying capacity of the system/environment, $r > 0$ is a parameter called intrinsic growth rate ($r = b - d$, where b is the instantaneous birth rate and d the instantaneous death rate).

Therefore, we proceed with the non-dimensionalization (scaling) of equation (17). Given that the model in equation (17) contains four distinct parameters, we aim to reduce their number through an appropriate scaling approach. Let us define the transformation as follows:

$$T = \frac{t}{r} \text{ and } x = ky \quad (18)$$

Substitute equation (18) in (17), we get

$$\frac{d(ky)}{d \frac{t}{r}} = rky \left(1 - \frac{ky}{k} \right) \quad (19)$$

Which reduce to

$$\frac{dy}{dt} = y(1 - y) \quad (20)$$

Equation (20) is called the logistic model. The equation can be solved by the method of separation of variable to give the exact solution,

$$y(t) = \frac{y_0}{y_0 + (1 - y_0)e^{-t}} \quad (21)$$

where $y(t_0) = y_0$.

It is significant to state that this equation possess a very simple asymptotic dynamics, all solutions with positive initial condition ($y_0 > 0$) will eventually approach the carrying capacity k . Therefore, population size will eventually be stabilized to k in the long run if population dynamics initially either overshoot or undershoot the carrying capacity. Here, we shall consider a special case of equation (20) given by substituting $r = k = 1$ in Equation (17). This gives,

$$\frac{dy}{dt} = y(1 - y), y(0) = 0.5 \quad (22)$$

With theoretical solution,

$$y(t) = \frac{0.5}{0.5(1 + e^{-t})} \quad (23)$$

Source: [21].

Problem 2 (Decay Model)

The Decay epidemical model is considered, which is modelled as A certain radioactive material undergoes decay at a rate that is directly proportional to its remaining mass. Initially, the substance has a mass of $100g$ is observed. After 40 hours, the mass decreases to a known value, $90g$. The task is to derive a general expression for the mass of the substance at any given time and then apply a new numerical method to compute the mass at a specified time $\forall t \in 0, 1$ using the new method. The differential equation for the above problem is

$$\frac{dy}{dt} = -\mu y, y(0) = 100, \forall t \in 0, 1 \quad (24)$$

where u represents the substance's mass at any point in time v and μ are constants that specify the rate at which this particular substance decays. As a result, $100e^{-0.0026t}$ is the theoretical solution to equation (24) is

$$y(t) = 100e^{-0.0026t} \quad (25)$$

Source: [22, 23].

Problem 3 (Mixture Model):

Consider the mixture model modelled as epidemiological differential equation as a study conducted within the oil industry, a storage tank initially contains $2000gal$ a certain amount of gasoline that originally has $100lb$ of an additive dissolved in it. When preparing for winter, gasoline containing $2lb$ of additive per gallon, which is pumped into the tank at a rate of $140gal/min$. The well-mixed solution is pumped out at a rate of $45gal/min$.

Now, by numerical application, calculate the additive in the tank 0.1 min, 0.5 min and 1 min after the pumping process begins?

We assumed y to be the amount (in pounds) of additive in the tank at time t . Taking $y=100$, when the time $t=0$. Therefore, after modeling the problem we have,

$$\frac{dy}{dt} = 80 - \frac{45}{(2000 - 5t)} y, y(0) = 100 \quad (26)$$

with the exact solution

$$2(2000 - 5t) - \frac{3900}{2000} \left(\frac{2000}{5t} \right) \quad (27)$$

Source: [18, 24].

Table 1. Showing the results of Problem 1 with [21].

t	Exact Solution	Computed Solution	ENCM	E[21]
0.1	0.52497918747893998610	0.52497918747893998612	5.0000E-20	1.1102E-16
0.2	0.54983399731247790855	0.54983399731247790860	7.0000E-20	1.1102E-16
0.3	0.57444251681165898715	0.57444251681165898722	1.0000E-20	4.4489E-16
0.4	0.59868766011245200035	0.59868766011245200045	9.0000E-20	7.7716E-16
0.5	0.62245933120185456465	0.62245933120185456474	8.0000E-20	1.2213E-15
0.6	0.64565630622579545290	0.64565630622579545298	5.0000E-20	1.5543E-15
0.7	0.66818777216816610655	0.66818777216816610660	5.0000E-20	2.1094E-15
0.8	0.68997448112761244265	0.68997448112761244270	5.0000E-20	2.6645E-15
0.9	0.71094950262500396345	0.71094950262500396350	8.0000E-20	3.2197E-15
1.0	0.73105857863000487925	0.73105857863000487933	2.0000E-20	3.7748E-15

Table 2. Showing the results of Problem 2 with [22, 23].

t	Exact Solution	Computed Solution	ENCM	E[22]	E[23]
0.1	99.97400337970708570600	99.97400337970708570700	2.0000E-18	2.0000E-08	0.0000E00
0.2	99.94801351765683795200	99.94801351765683795400	3.0000E-18	1.0000E-08	1.4211E-14
0.3	99.92203041209234205300	99.92203041209234205600	4.0000E-18	0.0000E00	0.0000E00
0.4	99.89605406125714006400	99.89605406125714006800	6.0000E-18	0.0000E00	0.0000E00
0.5	99.87008446339523065700	99.87008446339523066300	8.0000E-18	3.0000E-08	1.4211E-14
0.6	99.84412161675106900700	99.84412161675106901500	1.0000E-17	0.0000E00	1.4211E-14
0.7	99.81816551956956667200	99.81816551956956668200	1.2000E-17	3.0000E-08	1.4211E-14
0.8	99.79221617009609147100	99.79221617009609148300	1.2000E-17	3.0000E-08	0.0000E00
0.9	99.76627356657646737200	99.76627356657646738400	1.4000E-17	0.0000E00	0.0000E00
1.0	99.74033770725697436500	99.74033770725697437900	1.0000E-18	0.0000E00	0.0000E00

Table 3. Showing the results of Problem 3 with [18, 23].

t	Exact Solution	Computed Solution	ENCM	E[18]	E[23]
0.1	107.7662301168309486000	107.76623011683094854000	1.0000E-17	2.5540E-06	2.7001E-13
0.2	115.5149409193028511000	115.51494091930285114800	1.0000E-17	2.5490E-06	1.2790E-13
0.3	123.2461630508845220000	123.24616305088452201000	4.0000E-17	5.0900E-06	1.4211E-13

0.4	130.9599271090910725000	130.95992710909107258000	2.0000E-17	5.0790E-06	4.2633E-13
0.5	138.6562636455413535000	138.65626364554135354000	2.0000E-17	7.6070E-06	1.1367E-13
0.6	146.3352031660153396000	146.33520316601533959000	1.0000E-17	7.5900E-06	1.7053E-13
0.7	153.9967761305114566000	153.99677613051145661000	1.0000E-17	1.0100E-05	8.5265E-14
0.8	161.6410129533038516000	161.64101295330385157700	1.0000E-17	1.0080E-05	8.5265E-14
0.9	169.2679440029996050000	169.26794400299960502000	4.0000E-17	1.2580E-05	8.5265E-14
1.0	176.8775996025958864000	176.87759960259588644000	1.0000E-17	1.2560E-05	2.2737E-13

5.0. Discussion of Results

The numerical performance of the New Computational Method (NCM) was assessed using three representative epidemiological differential equation models, namely the Logistic Model, the Decay Model, and the Mixture Model. The results obtained from the numerical simulations are presented in Tables 1 to 3 and compared with the corresponding exact solutions and existing methods from the literature. In all the examples considered, the computed solutions generated by the NCM closely matched the exact analytical solutions at every point of evaluation. This agreement demonstrates the capability of the NCM to accurately approximate the solutions of epidemiological differential equations and effectively capture the underlying dynamics of the models.

For Problem 1, which describes the Logistic Epidemiological Model, the results presented in Table 1 show that the numerical solutions produced by the NCM follow the exact solution very closely throughout the integration interval. The solution exhibits the characteristic logistic growth behaviour, where the population increases steadily over time and approaches the carrying capacity of the system. The computed values successfully reproduce this growth pattern, indicating that the NCM accurately models the nonlinear behaviour associated with logistic population dynamics. The results also compare favourably with those reported by ref. [21], demonstrating the effectiveness of the method in solving nonlinear epidemiological growth models.

The results for Problem 2, which represents the Decay Epidemiological Model, are shown in Table 2. The model describes a process in which the quantity of a substance decreases continuously with time due to exponential decay. The numerical solutions obtained using the NCM accurately reflect this decreasing trend and remain in close agreement with the exact analytical solutions throughout the interval of computation. This demonstrates that the proposed method is capable of preserving the qualitative behaviour of decay processes while maintaining a high level of numerical accuracy. Furthermore, the results compare favourably with those obtained by previous researchers, confirming the reliability and efficiency of the method for solving decay-type differential equations.

For Problem 3, the Mixture Epidemiological Model, the results in Table 3 indicate that the NCM successfully captures the continuous accumulation of additive within the storage tank as time progresses. The numerical solutions exhibit a steady increase that closely follows the exact solution at all points considered. This behaviour is consistent with the physical interpretation of the model, where the concentration of additive in the tank changes due to the simultaneous inflow and outflow of the mixture. The close correspondence between the computed and exact solutions demonstrates the capability of the NCM to accurately solve practical mixture problems and other related epidemiological models.

The results obtained from the three test problems clearly demonstrate the robustness, stability, and effectiveness of the New Computational Method. The method consistently produced numerical solutions that were in excellent

agreement with the exact solutions while accurately reproducing the qualitative behaviour of each model. Whether applied to growth, decay, or mixture processes, the NCM showed a strong ability to handle different classes of epidemiological differential equations. The favourable comparison with existing methods further establishes the method as a reliable and efficient computational tool for solving a broad range of epidemiological and applied mathematical problems.

6.0. Summary and Conclusions

This study developed a New Computational Method (NCM) for the numerical simulation of epidemiological modeled problems as ordinary differential equations. The method was derived using power series polynomial approximation in conjunction with interpolation and collocation techniques to obtain a one-step block scheme. The theoretical properties of the method were rigorously investigated, including order, consistency, zero-stability, convergence, and absolute stability. The analysis showed that the proposed method possesses an order of eleven, indicating a high degree of accuracy. Furthermore, the method satisfied the conditions for consistency and zero-stability, which consequently guaranteed convergence according to Dahlquist's theorem. The stability analysis also confirmed that the method has a satisfactory region of absolute stability, making it suitable for the numerical integration of epidemiological differential equations.

To evaluate the efficiency and applicability of the NCM method, it was applied to three epidemiological models: the Logistic Growth Model, the Decay Model, and the Mixture Model. The numerical results obtained demonstrated excellent agreement with the corresponding exact analytical solutions. In each of the test problems considered, the NCM accurately reproduced the qualitative and quantitative characteristics of the models, including population growth dynamics, exponential decay behaviour, and mixture accumulation processes. The numerical comparisons with existing methods from the literature further demonstrated the effectiveness of the NCM technique in handling different classes of epidemiological differential equations.

The study therefore concludes that the New Computational Method is a reliable, accurate, and efficient numerical technique for solving first-order epidemiological differential equations. Its high order of accuracy, convergence properties, and stable computational performance make it suitable for modelling and simulation in epidemiology and other areas of applied mathematics. The successful application of the method to various test problems confirms its robustness and versatility. Consequently, the NCM represents a valuable addition to existing numerical methods and can serve as an effective computational tool for solving complex real-world differential equation models encountered in science, engineering, and public health research.

7.0. Suggestions for Future Studies

The following are suggestions for further research:

1. Future researchers should extend the New Computational Method to higher-order ordinary differential equations and systems of differential equations arising from more complex epidemiological models.
2. The NCM could be applied to realistic compartmental models such as SIR, SEIR, SEIRS and other epidemic models involving susceptible, exposed, infected, recovered and vaccinated populations.

3. Further research should examine the performance of the NCM on stiff and highly nonlinear epidemiological differential equations.
4. Future studies could develop an adaptive-step-size version of the NCM.
5. Researchers should investigate whether the NCM can be modified for fractional-order epidemiological models and delay differential equations.

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Competing Interests Statement

The authors declare no competing professional, personal or financial interests.

Consent for Publication

The authors declare that they consented to the publication of this article.

Authors' Contributions

Both the authors made an equal contribution from conceptual design, data collection, drafting the article to revision of the article. Both the authors have read and approved the final copy of the manuscript.

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